Physics with Black Hole Systems
of Known Mass at a Muon Collider

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12/12-12/16, 2004
Muon Collider Simulation Workshop, Miami Beach, Florida

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hep-ph/0411248
MUC-CONF-PHYSICS-299
Introduction

- **Unique features of a Muon Collider** $(\mu^+ - \mu^-)$:
  
  (Barger, Berger, Gunion, Han 1996)

  - Bremsstrahlung radiation effect is negligible (scales by $m_\mu^{-4}$)

  - Direct s-channel coupling to Higgs boson resonances is $40,000 \left(\frac{m_\mu}{m_e}\right)^2$ greater for $(\mu^+\mu^- \rightarrow h)$ than for $(e^+e^- \rightarrow h)$

  

  $\Rightarrow R_{h_{\text{higgs}}}^{(\mu^+\mu^-)} \gg R_{h_{\text{higgs}}}^{(e^+e^-)}$

  - Beam energy resolution is not limited by beamstrahlung smearing, such that the beam energy resolution $\sigma_{\sqrt{s}} \leq \Gamma_\text{total}_h$
Neutral Higgs Bosons Search

- At Muon Collider, there are 3 possible techniques for searching heavy neutral higgs bosons: $H^0$ (CP-even) and $A^0$ (CP-odd)

1. **Scan for $s$-channel Higgs** : $250 \text{ GeV} < m_{H^0/A^0} < 500 \text{ GeV}; \tan \beta > 3$

2. **Bremsstrahlung Tail** : Good for higher $\tan \beta$

3. **$H^0$, $A^0$ and $H^+H^-$ Production** : for very heavy Higgs bosons ($\sim 10 \text{ TeV}$)
   
   (We focus only the first two techniques)

- **Supersymmetry parameter** $\tan \beta \equiv v_2/v_1$, where $v_1, v_2$
  
  is the vacuum expectation values of neutral members of Higgs doublet of MSSM

- **In MSSM**, the heavy Higgs bosons are largely degenerate

  $\implies$ larger values of $\tan \beta$ heighten this degeneracy: ($\tan \beta = 5, 10$ and $20$)
Scan for $s$-Channel Higgs

- The Higgs boson cross section in $s$-channel:

$$\sigma_h(\sqrt{s}) = \frac{4\pi \Gamma(h \rightarrow \mu \bar{\mu}) \Gamma(h \rightarrow X)}{(s-m_h^2)^2 + m_h^2(\Gamma_h^\text{tot})^2} \implies \sigma_{\sqrt{s}} = (2 \text{ MeV}) \left( \frac{R}{0.003\%} \right) \left( \frac{\sqrt{s}}{100 \text{ GeV}} \right)$$

Separation of $A^0$ & $H^0$ by Scanning

- For the larger $\tan \beta$ the resonances are clearly overlapping with $\Gamma_{H^0,A^0} \sim 0.1 - 0.6$ GeV

- Muon Collider with sufficient energy resolution might be the only possible means for separating out these states: with $R = 0.01\%$ and $R = 0.06\%$
Black Holes Motivation

- We have observed Astronomical Black Holes (BH):
  
  Hubble uncovers dust disk around a massive Black Hole

  3,700 light-year-diameter dust disk

- The observable astronomical BH encourages us to explore
  miniature BH production in a Laboratory

- BH production in Lab could be the most promising signal of
  TeV-scale quantum gravity
In large extra dimensions at the TeV energy scale, Gravitons can propagate in the \( n = D - 4 \) extra dimensions.

The BH is characterized by the Schwarzschild radius

\[
 r_s = \frac{1}{\sqrt{\pi} M_{pl}} \left[ \frac{8\Gamma\left(\frac{n+3}{2}\right)}{(2+n)} \right]^{\frac{1}{n+1}} \left( \frac{M_{BH}}{M_{pl}} \right)^{\frac{1}{n+1}}
\]

- \( M_{pl} \sim \text{TeV} \) is fundamental Planck scale

If the impact parameter \( b < r_s \), \( \rightarrow \) an Event Horizon is formed.
Hawking’s Evaporation

- After Black Hole formed it will decay via Hawking evaporation process (Hawking radiation)

- The Black Holes emits into two modes:
  1. Along the brane (brane mode): Standard Model fields
  2. Into the extra dimensions (bulk mode): gravitons (invisible)

- Hawking radiation
• BH cross section can be estimated from the geometrical cross section (black disk)

\[ \sigma_{ij \rightarrow BH} \approx \pi r_s^2 = \frac{1}{M_{pl}^2} \left[ \frac{M_{BH}}{M_{pl}} \left( \frac{8\Gamma \left( \frac{n+3}{2} \right)}{(2+n)} \right) \right]^{\frac{2}{n+1}} \]

• LHC (proton-proton collider), we need to consider its cross section at the parton level (hampered by parton distributions)

\[ \sigma_{pp \rightarrow BH} \approx \sum_{ij} \int_{x_m}^{1} dx \int_{x}^{1} dy f_i(y, Q) f_j(x/y, Q) \sigma_{ij \rightarrow BH}(x, s, n) \]

○ \( x_m = M_{BH(min)}^2/s, \quad s = M_{pl}^2 \) and \( Q = \) the momentum transfer

○ \( f_i, f_j = \) Parton Distribution Function (PDF)

• For \( (e^+ - e^-) \) collider like CLIC, beamstrahlung smears the collision energy, and the NLC lacks the energy reach

• Muon Collider \( (\mu^+ - \mu^-) \) collider), the BH cross section is relatively simple

\[ \sigma_{\mu\mu \rightarrow BH} \approx \sigma_{BH}(s, n) \] (it does not depend on the minimum \( M_{BH} \))
BH Cross Section of 5 TeV ($e^+ - e^-$) collider at CLIC

Courtesy of Landsberg and Dimopoulos (hep-ph 0204031)

- The extra dimension $n = 4$
- The CM-energy $\sqrt{s} = 5$ TeV
- The beamstrahlung-corrected energy spectrum for CLIC machine
  is peaking at the nominal energy (5 TeV)
Cross Section at Muon Collider

- BH Cross Section for $4 \, TeV \, (\mu^+ - \mu^-)$

\[ \sigma_{BH} (4 \times 10^{-34} \, cm^{-2}) \]

- $n = D - 4$ extra dimensions

- $\sqrt{s} = 4 \, TeV$ CM-energy $\implies \sigma_{BH} \sim 7 \, nb$

- Using $\mathcal{L}_{\mu^+\mu^-} \sim 10^{33} (cm^{-2}s^{-1}) \implies$ BH production rate $\sim 7 \, BH/s$

$\implies \tau_{BH} \sim 10^{-27} \, s$
Temperature and Radius

- BH decay depends on Hawking temperature and is proportional to the inverse radius.

- Hawking temperature of (n+4)-dimensional BH:

\[
T_H = M_{pl} \left( \frac{M_{pl}}{M_{BH}} \frac{n+2}{8\Gamma\left(\frac{n+3}{2}\right)} \right)^{\frac{1}{n+1}} \frac{n+1}{4\sqrt{\pi}} \frac{1}{4\pi r_s}
\]

where \(r_s\) is Schwarzschild radius.

- The higher CM-energy (or the higher extra dimension), \(\Rightarrow\) the heavier and the colder BH.
• Horizon formation of CM-energy of 4 TeV with impact parameter $b$

![Graph showing horizon formation with impact parameter $b/b_{\text{max}}$ and different $n$ values.]

• The energy trapped by the horizon is a function of the impact parameter $b/b_{\text{max}}$

• For head-on collision, $\sim 60\%$ of CM-energy trapped by the horizon

• When extra dimension increases $\rightarrow M_{BH}/\sqrt{s}$ decreases

[We use Yoshino and Nambu's model (PRD 67, 2003)]
Total Missing Energy

- Total missing energy \(E_{\text{miss}}^{\text{Total}}\) provides a signature of un-observed gravitons and neutrinos that are emitted \(E_{\text{miss}}^{\text{Total}} = 2.7 \text{ TeV}\)
  - \(E_{\text{miss}}^{\text{Total}} = E_{\text{miss}}^{\text{Formation}} + E_{\text{miss}}^{\text{Evaporation}}\)
  - \(E_{\text{miss}}^{\text{Evaporation}} = \sum_i N_i E_i\)
  - \(N_i = \text{number of un-observed particles (neutrinos and gravitons)}\)
  - \(E_i = \text{its corresponding missing energy at evaporation process}\)
Transverse missing energy ($E_T$) is coming from the transverse momentum of neutrinos and gravitons ($E_T = 190$ GeV)

- Note that we assume all gravitons emit in brane modes
- $E_T$ distribution with different extra dimensions
BH Signatures

- What are the BH signatures experimentally?

1. Large cross section rate determined by the dimensionality of the extra dimensions (Giddings & Thomas 2002)
   \[ \implies \text{we have a relatively high cross section} \sim 7 \text{ nb} \]

2. Large missing energy (Cavaglia, Das & Maartens, 2003)
   \[ \implies \text{we have} \ E_{\text{miss}}^{\text{Total}} \sim 2.7 \text{ TeV} \]

3. Visible transverse missing energy (Giddings & Thomas 2002)
   \[ \implies \text{we have} \ E_T \sim 190 \text{ GeV} \]

4. The typical ratio of hadron to lepton in the evaporation process is 7:1

5. The typical ratio of hadron to photon is 60:1
Summary

- Muon Collider is a good place to study a direct s-channel Higgs boson, and to differentiate between $A^0$ and $H^0$

- Muon Collider at 4 TeV is a suitable place for producing miniature Black Holes with no beamstrahlung smearing

- Muon Collider provides a relatively high and constant cross section of BH

$$\sigma_{\mu\mu \rightarrow BH} \approx \sigma_{BH}(s, n)$$  (only depend on CM-energy and extra dimensions)

- $\sigma_{BH} \sim 7 \text{ nb}$, using $[\mathcal{L}_{\mu+\mu^-} \sim 10^{33}(cm^{-2}s^{-1})]$  
  $\implies$ BH production rate $\sim 7 \text{ BH/s } (\tau_{BH} \sim 10^{-27}s)$

- BH system (BH + gravitons) produced at rest with known mass

- Missing energy and transverse missing energy help us to explore gravitons, extra dimensions, Hawking radiation and quantum remnants