PHY-842: CLASSICAL ELECTRODYNAMICS II
Homework 1

1. In the proof of the existence theorem of the scalar potential, show that $\partial\psi/\partial x_2 = F_2$.

2. Give an example for nonvanishing vector and scalar potentials $\vec{A}(\vec{r},t)$ and $V(\vec{r},t)$ such that $\vec{E}(\vec{r},t) = \vec{0}$ and $\vec{H}(\vec{r},t) = 0$ for all $\vec{r}$ and t.

3. Consider an infinite plate positioned in the $y - z$ plane at $x = 0$. There is a charge q placed at the point $(d, 0, 0)$. Determine the charge distribution $\sigma(y, z)$ induced on the plate as well as the total force on the plate.

4. Consider the atomic nucleus with Z protons and atomic weight A as a uniformly charged sphere of radius $R = 1.5A^{1/3}$ fm (1 fm=10^{-13} cm). Find the maximum value of the electric field of the nucleus (express in Volt/m).

5. Let q_1 and q_2 be two opposite charges on the x axis at l_1 and l_2. Prove that there is a sphere on which the total potential of the two charges vanishes.

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Homework 2

1. A point-like electric dipole $\mathbf{d}$ is oriented in such a way that it has components $(d_x, d_y, 0)$ and placed in vacuum at a distance $x = a$ from an infinite conducting plane yz. Calculate the force acting on the dipole.

2. Two coaxial cylinders with radii R_1 and R_2 and length $L >> R_{1,2}$ are charged with charges q_1 and q_2. Determine the electrostatic potential of the system, and observe that ϕ is linear in q_1 and q_2. Determine the capacitance matrix and the capacitance of the system in case $q_1 = -q_2$.

3. A point charge q is placed inside an angle α_0 formed by two grounded conducting planes. Find the electric field inside the angle for $\alpha_0 = 45^\circ$, 60° and 90°.

4. The centers of three identical small conducting spheres of radius r are forming an equilateral triangle with the side length $l \gg r$. Initially each sphere carries a charge q. Then each sphere, in turn, is grounded for a sufficiently long time. Determine the remaining charges q_1, q_2, q_3.

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Homework 3

1. Two conductors are separated by a distance r which is much greater than their sizes. Their capacitances are equal to $C_{11} \equiv C_1$ and $C_{22} \equiv C_2$. The conductors are kept under potentials φ_1 and φ_2. Determine the potential in space and the force acting between the conductors.

2. A closed conducting surface with the potential φ_1 contains inside another conductor with the potential φ_2. A measurement of the potential at some point P between the conductors gave the value φ_P. After that both conductors are grounded, and an external charge q is placed at the point P. What charges are now induced at the conductors?

3. An infinite thin metallic wire is uniformly charged over its length with the linear charge density χ. The wire is located in a material of dielectric constant ε, and is parallel to the axis of a conducting round cylinder of radius R; the distance between the wire and the cylinder axis is $a > R$. Find the electric potential in space.

4. Calculate the force experienced by a charge q residing in vacuum and placed a distance d from a infinite wall of dielectric constant ε.

5. Three wedges (infinite in the z-direction) are fit together to cover completely the xy-plane. The angles between the wedges, $\alpha_1, \alpha_2, \alpha_3$ ($\alpha_1 + \alpha_2 + \alpha_3 = 2\pi$), are filled with dielectrics with dielectric constants $\epsilon_1, \epsilon_2, \epsilon_3$, respectively. The charge q is placed at the origin. Determine the electrostatic potential and vectors $\vec{\mathcal{E}}$ and $\vec{\mathcal{D}}$ in all three regions.

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Homework 4

1. Find the force per unit area f of attraction between the plates of a plane capacitor (area of plates S, distance between them d) if
   a. the empty capacitor is connected to a battery of voltage V;
   b. the capacitor is charged to a voltage V, then separated from the battery and subsequently completely filled with a liquid dielectric with dielectric constant ϵ;
   c. the liquid of point b is substituted by a piece of a solid dielectric with the same ϵ which does not touch the plates having the size a little smaller than d;
   d. the capacitor is first filled with the liquid of point b and then charged to the same voltage V;
   e. the solid dielectric of point c is placed inside, and then the capacitor is charged to the voltage V.

2. A conducting sphere of radius R is floating on the plane interface between two liquid dielectrics (dielectric constants ϵ_1 and ϵ_2); the center of the sphere is at the boundary. Find the electrostatic potential, electric field, electric induction, polarization of the dielectrics and the distribution of charges if the sphere carries the free charge q.

3. A voltage V is applied between the inner and the outer cylinders of a cylindrical capacitor (radii a and b, respectively). The capacitor has been inserted by its lower end into a dielectric liquid of mass density ρ_m, which has raised its level inside the capacitor by the height h. Determine the dielectric constant of the liquid.

4. A sphere of radius R carries the total electric charge q. The sphere is immersed into a dielectric medium with the dielectric constant $\epsilon(r)$ depending on the distance r from the center of the sphere. Find electric displacement, electric field, electrostatic potential, polarization, volume and surface distribution of bound charges if
   a. the sphere is made of a metal;
   b. the sphere is made of a dielectric with the dielectric constant ϵ_0, and the free charge q is uniformly distributed over its volume.
   Apply your general results to a specific case $\epsilon(r) = 1 + a/r$.

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Homework 5

1. A square grid consists of $3 \times 3 = 9$ identical square cells made of a uniform metallic wire. A resistance of each link is equal to r. A current enters a corner of the grid and comes out of the opposite corner. Find the resistance R of the whole grid.

2. Two dielectric plates (thicknesses d_1 and d_2, dielectric constants ϵ_1 and ϵ_2) are inserted between the plates of a plane capacitor under voltage V in such a way that they tightly fit the distance $d = d_1 + d_2$ between the plates of the capacitor. The dielectrics are not perfect but their conductivities λ_1 and λ_2 are much smaller than the conductivity of the plates of the capacitor. Find the electric field, electric induction, current density and the distribution of free and bound charges inside the capacitor.

3. A spherical cavity of radius R is made inside a macroscopic conductor. The conductor is kept under the potential V. The cavity is filled with the dielectric (dielectric constant ϵ). A point charge q is placed inside the cavity at a distance a ($a < R$) from the center. Determine the electrostatic potential in the cavity.

4. Calculate the capacitance of a system of two long parallel wires of round cross section (radius a, distance between the centers of the wires $d \gg a$, length $L \gg a, d$). Define capacitance as $C = q/V$ where the total charges of the wires are q and $-q$, and the corresponding difference of their potentials is V.

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Homework 6

1. Consider the set-up of the Problem 2, Homework 5. .

   a. Let the voltage V be applied at the initial moment $t = 0$. Find the interlayer surface density σ_0 and the electric fields in both parts of the capacitor as functions of time. Show that after an exponentially decaying transitional process the capacitor reaches the steady state.

   b. Express the characteristic time of the transitional process in terms of the capacitances and resistances of the two parts of the capacitor. Show that the voltage is divided between the two sections of the capacitor according to their capacitances in the beginning of the process and according to their resistances in the final stationary state.

   c. Is it possible to find the parameters which would guarantee the time independent division of the potential during the entire process?

2. Two long round parallel wires (radius $R = 1.5$ cm) carry equal electric currents I uniformly distributed over the areas of their cross sections. The distance between the axes of the wires is $l = 5$ cm. Consider a line crossing the axes of the wires perpendicular to them. Find the points on this line (if any), where the total magnetic field vanishes (consider the cases of the currents in the same direction and in the opposite direction).

3. *a.* An infinite straight current I going in the y-direction is located in vacuum at distance a from the flat surface of the uniform magnetic medium (magnetic permeability μ) which occupies the semispace $x < 0$. Calculate vector potential, magnetic induction $\vec{\mathcal{B}}$ and magnetic field $\vec{\mathcal{H}}$ in both parts of space; magnitude and direction of the force acting onto the wire per unit length. Use the method of imaging.

   b. Semispaces $x > 0$ and $x < 0$ are filled by homogeneous magnetic media with permeabilities μ_1 and μ_2, respectively. A plane contour C, of arbitrary shape, with current I is placed in the first medium parallel to the boundary at distance a from it. Write down the expression for the vector potential in both media in terms of contour integrals.

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Homework 7

1. A metallic torus (magnetic permeability μ) is uniformly and densely covered by N loops of helix winding of thin wire with current I. A radius of the cross section of the torus is a, the distance from the center of the cross section to the axis of the torus is b.

   a. Find the magnetic field inside the torus and the magnetic flux through the cross section neglecting the scattered flux outside the torus.

   b. Calculate the self-inductance of the torus. Consider the limit of $b \gg a$ and explain the physical meaning of the result in this limit.

   c. The secondary isolated helix winding of N' turns is placed on top of the primary toroidal coil of N turns. Find the mutual inductance of the coils.

2. A long cylindrical coaxial cable with the axis along the z-direction consists of a round wire of radius a enclosed by a second wire which is a coaxial hollow cylinder (inner radius b, outer radius d, $d > b > a$). The first and the second wire, both with conductivity λ, carry steady currents I and $-I$, respectively. The currents are uniformly distributed over the corresponding cross section areas.

   a. Calculate the electric field, electrostatic potential in all regions inside the cable, and density of free and bound charges on the boundaries if the insulator between the wires ($a > r > b$) has the dielectric constant ϵ. The voltage between the wires equals V at $z = 0$.

   b. Determine the magnetic field and magnetic induction in all regions inside the cable (the magnetic permeability of the wires is μ_1 and the magnetic permeability of the insulator is μ_2).

   c. Calculate the self-inductance per unit length of the cable.

3. A contour with current I_1, self-inductance L and resistance R is put from rest into motion (without deformation) in the magnetic field of the fixed steady current I_2. Show that the mechanical work δW performed by the magnetic forces during a small time interval δt is negative if the magnitude of the current I_1 is increased (*Lenz rule*).

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Homework 8

1. Consider two infinite slabs of thickness 1m oriented in $y - z$ direction with a permeability μ and conductivity σ. The slabs are separated by empy space of 1m. To the right of the two slabs, there is a magnetic field of the form $\vec{H}(x, y, z, t) = H_0 \cdot \cos(\omega t) \cdot \vec{e}_x$. Determine $\vec{H}(x, y, z, t)$ on the left of the two slabs.

2. A uniform external magnetic field $\vec{\mathcal{H}}_e$ is applied to a superconducting sphere of radius a at temperature $T < T_c$.

   a. show that outside the sphere

$$\mathbf{B} = \vec{\mathcal{H}}_e - \nabla \frac{(\mathbf{m} \cdot \mathbf{r})}{r^3}, \tag{212}$$

   where r is the distance from the center of the sphere, and find the direction and magnitude of the vector $\mathbf{m}$. Hint: think about the problem of the metal sphere in an external electric field.

   b. if $\mathcal{H}_c$ is the critical magnetic field of the bulk material, at what magnitude of the external field the superconductivity of the sphere starts to be destroyed?

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Homework 9

1. Determine the Impedance of system 1.

2. Determine the Eigenfrequencies of circuit 2, and interpret their meaning.

3. Determine the relationship of U_2 on ω in system 3.

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Homework 10

1. Assume a conducting wire is described by the curve $\vec{\gamma}(s)$, where $s \in [a, b]$. Suppose the wire carries current I and is moving in a magnetic field of magnitude $\vec{B}(\vec{r})$. Show that the force on the wire is given as

$$\mathbf{F} = \frac{I}{c} \int_a^b d\mathbf{l} \times \mathbf{B} \tag{220}$$

2. A heavy horizontal rod ab of mass m and length l is sliding down vertical directing rods aa' and bb' in a uniform magnetic field B perpendicular to the plane $a'abb'$. Assuming that at the initial moment the rod was at rest, find its velocity as a function of time if

   a. the directing rods are closed by a resistance R;

   b. the directing rods are closed by a coil with a resistance R and inductance L in parallel;

   c. the directing rods are closed by a capacitor C.

   Neglect resistances and inductances of the rods.

3. Derive Snell's law for the transition of a wave from one medium of index of refraction n_1 into another medium of index of refraction n_2.