Injection, Extraction, and 6D Muon Cooling with an Anti–Cyclotron

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Physics with Black Hole Systems of Known Mass at a Muon Collider

- Only the Muon Collider can produce black hole systems (black hole plus initial gravitons) of known mass.
- Known mass could be critical in measuring: Quantum Black Hole Remnants, Scanning production turn on, Initial/final gravitons as missing energy.
- CLIC $e^+e^-$ suffers from beamstrahlung 5 TeV spectrum from Greg Landsberg
Uranium energy (MeV/u) 10  
Relativistic Energy Limit 100  
Min. q/A (for 10 MeV/u) 0.15  
$B_{\rho_{\text{max}}}$ (kG-cm) 3018  
E Constant, K (E = $q^2/A$) 440  
Max. magnetic field (kG) 16.0  
Magnet fraction (52° hills) 0.58  
Number of sectors 4  
Injection E, U ion (MeV/u) 0.6  
Radius Ratio ($R_f/R_i$) 3 – 4.3  
Extraction mean R (m) 3.15  
RF freq. range, (MHz) 6 – 14  
Magnet weight, tons 2300

Peak V, fundamental, kV 250  
2nd Harmonic V, % fund. 26  
Power, fundamental, kW 400  
Power, 2nd harmonic, kW 100  
Resonator Length, m 8.6  
Resonator dia. (max.), m 3.3  
Amplitude Stability 1 in $10^4$  
Phase stability, deg ±0.1  
Energy Ratio ($E_f/E_i$) 9 – 19  
Injection $R$, $R_f/R_i=3$ (m) 1.05  
RF freq., 10 MeV/u (MHz) 13.22  
Harmonic # (10 MeV/u U) 6
Tabletop 6D Cooling Rings with RF

- **6 Sector Dipole Ring**
  Weak edge focusing, $B = 5.2$ Tesla
  Add skew quads to mix x and y
  Radius = 30 cm, 250 MeV/c muons
  45 MeV/m 201 MHz RF
  100 atm $H_2$ everywhere
  Merit factor = 400 after 250 orbits

- **12 Cell FFAG Ring**
  Strong Focusing, $B = 2.6$ Tesla
  $+30^0$ and $-15^0$ Bends
  Radius = 96 cm, 250 MeV/c muons
  8 MeV/m 201 MHz RF
  100 atm $H_2$ everywhere
  Merit factor = 120 after 50 orbits
Tabletop 6D Cooling Rings with RF

- **Principles**
  - Simulate with ICOOL and SYNCH
  - Transverse ionization cooling
  - Higher energy $\rightarrow$ Longer path
  - $\rightarrow$ Emittance exchange
  - $\rightarrow$ Longitudinal cooling
  - Constant $\beta$ allows $\text{H}_2$ everywhere
  - High pressure $\text{H}_2$ inhibits RF breakdown

- **Immediate Goal**
  - Build 6 sector Demonstration Ring
  - 1.5 Tesla
  - Merit factor $= 10$
Anti-cyclotron, NIM A278 (1989) 368
- Magnet 3 Tesla
  4 concentric coils
  Weak focusing
  Azimuthally symmetric field

- \( \frac{dE}{dx} \)
  Injection radius = 120 mm,
  \( p = 105 \) MeV/c, 0.3 mbar hydrogen

- Anti-protons adiabatically spiral to the center. \( \frac{dE}{dx} \) cannot be too high.
bullet Final anti-proton swarm. KE = 2 keV
  r = 15 mm
  h = 40 mm

bullet 20 microsecond spiral time for 0.3 mbar
bullet 1 microsecond spiral time for 10 mbar

bullet Pulsed electric kicker in Z
  80 ns pulse, 20 ns rise
  500 V/cm kick
  Anti-protons moves 32 cm in 500 ns
  Lighter muons will go farther

bullet A long bunch train is coalesced into one swarm
• Cyclotron frequency
  \[ f = \frac{\omega}{2\pi} = \frac{qB}{2\pi m} \]

• \( \frac{f_p}{f_\mu} = \frac{938}{106} = 8.8 \)
  1 \( \mu \)S spiral \( \rightarrow \) 0.11 \( \mu \)S spiral
Muon Swarm Size Estimate

- Put $10^{12}$ muons at a point
  Take $B = 2.9$ Tesla
  Set electric repulsion = Lorentz force
  Find radius
  Estimate, not orbit!!!

- $E = vB; \ v = qBr/m$
  
  $10^{12} \ q / (4\pi\varepsilon_0 \ r^2) = q \ r \ B^2 / m$
  
  $r = \left[10^{12} \ m / (4\pi\varepsilon_0 \ B^2)\right]^{1/3}$
  
  $= 6\text{mm}$

- Put a wire through the muon swarm
  Neutralize the charge!

- $10^{12}$ electrons move in a $\mu\text{S} \rightarrow 1 \text{ Amp}$
Beam Pipe

- The central region of the anti-cyclotron may need an Al or Be beam pipe.
- Lower gas pressure so muons won’t range out before they spiral all the way in.
- Lower gas pressure so a lower electric field can kick a swarm out after it has built up.
- Slow muons like to stick to hydrogen.
- Three positive muon solutions
  Positively charged foils instead of gas.
  Helium gas to inhibit muonium formation.
  Let muonium form. Laser disassociate.
- Two negative muon solutions
  Negatively charged foils instead of gas.
  Deuterium-Tritium gas. Fusion frees muons.
Imagine a 250 MeV/c muon entering an 0.5 Telsa ring tangent to a 1688 mm radius. A small kick gets it in.

Now imagine that a swarm needs to spiral in 500 mm and lose 30% of its momentum in one orbit.

KE loss needed = 67 MeV in 9000 mm

LiH absorbs 160 MeV/m → 420 mm
LH$_2$ absorbs 30 MeV/m → 2300 mm

So its worth simulating dE/dx injection.
$R_{in} = 1668 \text{ mm}$

Gas density $= 50 \text{ atm}$

$B_y = 0.5 \text{ Tesla}$

$B_x = (y/5) \sin \theta$

$B_z = (y/5) \cos \theta$

$P_\mu = 250 \text{ MeV/c}$

$\frac{dE}{dx} \text{ Injection}$
Busch’s Theorem and Ejection

- Accelerator Physics and Engineering
  A. W. Chao and M. Tigner, page 101

- \[ \dot{\phi} = \left[ \frac{e}{2\pi \gamma m} \sqrt{r^2(s)} \right] [\Phi(s) - \Phi_k] \]
  \( \dot{\phi} \) is the azimuthal angle
  \( r(s) \) is the radius of the beam edge
  \( \Phi(s) \) is the magnetic flux \( (\pi r^2 B_s) \) at s
  \( \Phi_k \) is the flux \( (\pi r^2 B_k) \) at the cathode

- \( L_z = r^2 \gamma m \dot{\phi} = -eB \frac{r^2}{2} = xp_y - yp_x \)
- \( L_z = 0.3 \ (0.5 \ \text{Telsa}) \ (0.10 \ m)^2 \ / \ 2 = .0008 \)
- \( L_z = 800 \ \text{MeV/c} \cdot \text{mm} \)
- \( L_z = 8 \ \text{MeV/c} \times 100\text{mm} \)

- An 8 MeV/c kick is moderately large.
- So, maybe increase the mirror ratio...
Damped Harmonic Oscillator

- Generalized Angular Momentum
  \[ L_g = L_z - e r A_\theta, \]
  NIM A278 (1989) 368

- Quasipotential Well,
  \[ \eta = e/M \]
  \[ U(r,z) = V(r,z) - (1/2\eta r^2) \left( L_g/M + \eta r A_\theta \right)^2 \]

- (a) \( U'(r,0) \) [MeV] vs \( r \) [mm] for various \( L_g \)
  a, b, c, and d are stable orbit radii

- (b) \([U'(r_0,z) - U'(r_0,0)] \) [MeV] vs \( z \) [mm] for various \( r_0 \)
• A muon spirals in 50 atm of H$_2$. 
• Vertical damping from 10 to 7 cm as a 300 MeV/c muon spirals in (50 atm H$_2$) from 100 cm using non-Maxwellian fields. $B(y,x,z) = (1, (y/5) \sin \theta, (y/5) \cos \theta)$ Tesla
A New Method to Produce Negative Muon Beam of keV energies

- Foil: 3 nm of nickel (3 \( \mu g/cm^2 \)) on Formvar
- 30 min. of sputtering – Franz Kottmann

Cyclotron trap at PSI

\[ 10^5 \text{ } \mu \text{A/s} \@ 20\ldots50 \text{ keV} \]

scale by 10^6 \rightarrow N = 10^{11} /s

\[ v = 1.5 - 30 \text{ cm/s} \]

Fig. 2. Principle of the extraction method.
Emittance Reduction Goals

- $\epsilon = (\Delta p_x \Delta x) (\Delta p_y \Delta y) (\Delta p_z \Delta z)$
- $\Delta p_x$: 50 MeV/c → 1 MeV/c
- $\Delta p_y$: 50 MeV/c → 1 MeV/c
- $\Delta p_z$: 50 MeV/c → 1 MeV/c
- $\Delta x$: 150 mm → 100 mm
- $\Delta y$: 150 mm → 100 mm
- $\Delta z$: 10000 mm → 100 mm

Re-accelerate. Exchange $\Delta p_x$ for $\Delta x$ and $\Delta p_y$ for $\Delta y$. Repeat Anti-Cyclotron

- $\Delta p_x$: 15 MeV/c → 1 MeV/c
- $\Delta p_y$: 15 MeV/c → 1 MeV/c
- $\Delta p_z$: 1 MeV/c → 1 MeV/c
- $\Delta x$: 50 mm → 30 mm
- $\Delta y$: 50 mm → 30 mm
- $\Delta z$: 100 mm → 30 mm