Optimal regime for a $\mu^+\mu^-$ collider

• All muons should be collected in a single bunch, because at given number of muons, luminosity is inversely proportional to the number of bunches.

• The bunch should be compressed longitudinally as fast as possible to be accepted by a high-frequency (high voltage) cooling system.

• A cooling is required at the compression because particles populate a very diffuse phase space, and initial beam length and energy spread should be large to capture more muons.
• **PD** – Proton Driver: 24 GeV.

• **TS** – Target Station: mercury jet in $20 \to 4.4$ T solenoid.

• **PR** – Phase Rotation channel: $20 \text{ m, } 4.4 \to 1.75$ T, 36.37 MHz, 6.37 MeV/m, $+10 \text{ m drift.}$

• **BC** – Bunch Compressor: ring cooler 72.3 m, 1.75 T, 36.37 MHz, 6.37 MeV/m LH$_2$ absorbers, LiH wedge absorbers.

• **MS** – Matching Section: 14 m, 1.75 $\to$ 3.5 T, 203.4 MHz, $7 \times 4.8$ MV cavities.

• **RF** – RFOFO ring cooler: 33 m, $\pm 2.74$ T, 203.4 MHz, 16 MeV/m, LH$_2$ wedge absorbers.

• **LL** – Li Lens cooling channel: 92 m, $\sim 10$ T Li lenses and solenoids, 201.25 MHz, 12-14 MV/m$^2$. 
Conclusion: Phase rotation channel should be tuned on $p \approx 200$ MeV/c.
Phase Rotation - Decay Channel

• It is reasonable to use the same RF both for phase rotation and bunch compression.
• Possible frequencies are $f = 3.637 \times h$ MHz:
  - $32.74, 36.37, 40.01, 43.65$ ...
• Presumed accelerating gradient
  $$V' = 2\sqrt{n} \text{ MV/m}.$$
• Optimal frequency is $36.37$ MHz ($h = 10$)
  $$V' = 6.37 \text{ MV/m}.$$
• Lower frequency: more capture – slower cooling – more decay.
• Higher frequency: less capture – faster cooling – less decay.
• Channel: 20 m RF + 10 m drift = 30 m.
• Solenoid: 4.4 $\rightarrow$ 1.75 T on first 5 m.
Beam at the Phase Rotation

Long. space after ph–rotation (red – pions, blue – muons)

Number of particles in bands
\[ \Delta E = 100 \ (10) \ 140 \text{ MeV} \]

Efficiency of the phase rotation
\[ ((\mu+\pi)/p \text{ at } \Delta E=120 \text{ MeV, } L = 5 \text{ m}) \]
Bunch Compressor

Nominal energy: 220 MeV
Circumference: 72.291 m
Bending radius: 52 cm
Bending field: 1.238 T
Normalized field gradient: 0.5
Length of short SS: 1.948 m
Length of long SS: 6.68 m
Short solenoid max field: 2.35 T
Long solenoid field: 1.75 T
Revolution frequency: 3.637 MHz
Accelerating frequency: 36.37 MHz
Accelerating gradient: 6.4 MeV/m
Synchronous phase: 30 deg
LH main absorber, length: 54.5 cm
LiH wedge absorber, dE/dy: 1 MeV/m

Beam cooling

Phase density of the beam
(parameter – grad. of w. absorber)

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<thead>
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<th>Density</th>
<th>Density</th>
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<td>0.02</td>
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<tr>
<td>0.04</td>
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<table>
<thead>
<tr>
<th>Long. dens. (µ/p/cm)</th>
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<tbody>
<tr>
<td>0.9 MeV/cm</td>
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<tr>
<td>1 MeV/cm</td>
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<tr>
<td>1.1 MeV/cm</td>
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</table>

<table>
<thead>
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<th>6D dens./10 (µ/p/cm³)</th>
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<td>0.00</td>
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<tr>
<td>0.01</td>
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<td>0.03</td>
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<tr>
<td>0.04</td>
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</tbody>
</table>
Long. Phase Space at Compression

Horizontal axis – ct (cm), vertical – total energy (MeV). Number of turns is shown in each figure.
Matching Section

- The matching is required first of all because FR changes from 36.37 to 203.4 MHz.
- Axial magnetic field should increase from 1.75 to 3.5 T.
- 14 m long matching section contains 7 cavities 203.4 MHz, 4.8 MV.
- Magnetic field increases linearly.

Longitudinal phase space
RFOFO Ring Cooler

Coil radius 77/88 cm
Coil length 50 cm
Coil tilting 52 mrad
Current density 95.27 A/mm²
Ac. frequency 203.4 MHz
Ac. gradient 16 MeV/m
Synch. phase 33 deg
Absorber (LH) 12.5 MeV, 1 MeV/cm

Beam parameters at cooling

After 10 turns:
Hor. emit. 2.6 mm
Vert. emit. 2.4 mm
Long. emit. 3.2 mm
Yield 0.054 μu/p
Lithium Lens Cooler

<table>
<thead>
<tr>
<th>Element</th>
<th>Length (cm)</th>
<th>Radii (cm)</th>
<th>J (A/mm$^2$)</th>
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<tbody>
<tr>
<td>Li lens</td>
<td>13.45</td>
<td>0/3</td>
<td>355.0</td>
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<tr>
<td>High-field coil</td>
<td>22.45</td>
<td>6/14</td>
<td>84.62</td>
</tr>
<tr>
<td>Low-field coil</td>
<td>206.66</td>
<td>69/71</td>
<td>81.69</td>
</tr>
</tbody>
</table>
Li Lens Cooler: Beta-function

β at the lens center vs energy
(blue – hard edge approximation)

β-function vs z

Cell

Li lens
Li Lens Cooler: Simulation

Evolution of the beam parameters at the Li lens cooler

After 35 cells:

- Trans. emittance (mm) 0.56
- Long. emittance (mm) 9.3
- Yield (muon/proton) 0.047
Conclusion

• Single muon bunch containing about 0.05 muons per incident proton can be obtained by means of described system.

• Its transverse emittance is about 0.6 mm what is still large for a muon collider.

• More transverse cooling is difficult because of longitudinal heating in the Li lens channel.

• Another serious restriction is high field matching solenoid. Longer Li lenses should be applied to use their ends with less gradient for the matching.