Parametric Resonance Cooling – Simulations

Kevin Beard* and Alex Bogacz*

in collaboration with

Slava Derbenev* and Rol Johnson♦

*Jefferson Laboratory

♦ Muons Inc.

2004 Workshop on Muon Collider Simulations, Miami Beach, FL December 15, 2004
Overview

- Final transverse ionization cooling - Parametric resonance enhancement
- Resonant transport channels for final cooling - lattice prototypes
  - quadrupole based
  - solenoid based
- Transverse beam dynamics in the cooling channel – tracking studies
  - ‘soft-edge’ solenoid
    - liner transfer matrix
    - nonlinear corrections (in tracking)
  - thin ‘ideal’ absorber model
- G4BL simulation of a solenoid channel with absorbers – basic tools
  - Hydrogen absorber
  - Solenoid transfer matrix from G4BL simulation - symplectification
Transverse parametric resonance cooling

- Transport channel (between consecutive absorbers) designed to replenish large angular component, $x'$, sector of the phase-space, ‘mined’ by ionization cooling process.

- **Parametric resonance** in an oscillating system - perturbing frequency is equal to the harmonic of the characteristic (resonant) frequency of the system.

- Normal elliptical motion of a particle’s transverse coordinate in phase space becomes hyperbolic – resulting beam emittance has a wide spread in $x'$ and narrow spread in $x$ – sector of the phase-space where ionization cooling is most effective.
Transfer matrix of a periodic resonant lattice

- Symplectic transfer matrix, $M(s)$, for a beamline (in x or y)

$$M(s) = \begin{bmatrix} e^{-\lambda s} \cos \psi & g \sin \psi \\ -\frac{1}{g} \sin \psi & e^{\lambda s} \cos \psi \end{bmatrix}$$

- Lattice period can be designed in such a way that $\sin \psi = 0 \Rightarrow \psi = n\pi$, $n = 1, 2$.

$$M(s) = \begin{bmatrix} e^{-\lambda s} \cos \psi & g \sin \psi \\ -\frac{1}{g} \sin \psi & e^{\lambda s} \cos \psi \end{bmatrix} \rightarrow \begin{bmatrix} e^{-\lambda s} & 0 \\ 0 & e^{\lambda s} \end{bmatrix}$$

- Coordinate and angle are uncoupled - resulting beam emittance has a wide spread in $x'$ and narrow spread in $x$.

$$x_{s_0+s} = \pm e^{-\lambda s} x_{s_0} \quad x \times x' = \text{const}$$

$$x'_{s_0+s} = \pm e^{\lambda s} x'_{s_0}$$
Resonantly perturbed uniform triplet lattice (H)
Resonantly perturbed uniform triplet lattice (V)
Symmetrized double cell ($\Delta\phi_x = 3\pi = \Delta\phi_y$)
4-cell resonant channel

- Uniform triplet lattice resonantly perturbed by a singlet
- Absorber placed half way between triplets
Final cooling – initial beam parameters

\( p = 287 \text{ MeV/c} \)

<table>
<thead>
<tr>
<th></th>
<th>( \varepsilon_{\text{rms}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>after helical cooling channel</strong></td>
<td></td>
</tr>
<tr>
<td>normalized emittance: ( \varepsilon_x/\varepsilon_y )</td>
<td>mm·mrad</td>
</tr>
<tr>
<td></td>
<td>300</td>
</tr>
</tbody>
</table>
| longitudinal emittance: \( \varepsilon_L \)  
\( (\varepsilon_L = \sigma_{\Delta \rho} \sigma_z/m_{\mu c}) \)  
momentum spread: \( \sigma_{\Delta \rho/\rho} \)  
bunch length: \( \sigma_z \) | mm                           |
|                                 | 7                             |
|                                 | 0.03                          |
|                                 | 100                           |
Angular ‘shearing’ of the transverse phase-space

View at the lattice

View at the lattice
Solenoid cell

**Diagram 1:**

- Beta_X vs. Beta_Y
- Disp X vs. Disp Y

**Diagram 2:**

- Phase vs. Q_X
- Q_Y vs. Q_Y

**Parameters:**

<table>
<thead>
<tr>
<th>Code</th>
<th>Length [cm]</th>
<th>Magnetic Field [kG]</th>
<th>Aperture [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>c1</td>
<td>130</td>
<td>38.1</td>
<td>10</td>
</tr>
<tr>
<td>c2</td>
<td>80</td>
<td>-34.3</td>
<td>10</td>
</tr>
</tbody>
</table>
'soft-edge' solenoid model

Zero aperture solenoid - ideal linear solenoid transfer matrix:

\[
M_{\text{sol}} = 
\begin{bmatrix}
  \frac{1 + \cos(kL)}{2} & \frac{\sin(kL)}{k} & \frac{\sin(kL)}{2} & \frac{1 - \cos(kL)}{k} \\
  -\frac{k \sin(kL)}{4} & \frac{1 + \cos(kL)}{2} & -\frac{1 - \cos(kL)}{k} & \frac{\sin(kL)}{2} \\
  -\frac{\sin(kL)}{2} & -\frac{1 - \cos(kL)}{k} & \frac{1 + \cos(kL)}{2} & -\frac{\sin(kL)}{4} \\
  \frac{k \left(1 - \cos(kL)\right)}{4} & -\frac{\sin(kL)}{2} & -\frac{\sin(kL)}{4} & \frac{1 + \cos(kL)}{2}
\end{bmatrix}
\]

\[k = eB_0/pc\]
‘soft-edge’ solenoid – edge effect

Non-zero aperture - correction due to the finite length of the edge:

- It decreases the solenoid total focusing – via the effective length of:

\[ L = \frac{1}{B_0} \int_{-\infty}^{\infty} B_z(s) \, ds \]

- It introduces axially symmetric edge focusing at each solenoid end:

\[ \Phi_{\text{edge}} = \frac{1}{2} \left( \int_{-\infty}^{\infty} B_z^2(s) \, ds - B_0^2 L \right) = -\frac{k^2 a}{8} \]

\[ k = eB_0/pc \]

- axially symmetric quadrupole

\[ \mathbf{M}_{\text{soft sol}} = \mathbf{M}_{\text{edge}} \mathbf{M}_{\text{sol}} \mathbf{M}_{\text{edge}} \]

\[ \mathbf{M}_{\text{edge}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\Phi_{\text{edge}} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\Phi_{\text{edge}} & 1 \end{bmatrix} \]
‘soft-edge’ solenoid – nonlinear effects

Nonlinear focusing term $\Delta F \sim \mathcal{O}(r^2)$ follows from the scalar potential:

Scalar potential in a solenoid

$$\phi(r,z) = \phi_0 + B_z z + \left( \frac{d}{dz} B_z \right) \frac{2 \cdot z^2 - r^2}{4} + \left( \frac{d^2}{dz^2} B_z \right) \frac{z \cdot (2 \cdot z^2 - 3 \cdot r^2)}{12} + \left( \frac{d^3}{dz^3} B_z \right) \frac{8 \cdot z^4 - 24 \cdot z^2 \cdot r^2 + 3 \cdot r^4}{192}$$

Solenoid B-fields

$$B_z(r,z) = B_z - \left( \frac{d^2}{dz^2} B_z \right) \frac{r^2}{4}$$

$$B_r(r,z) = -\frac{r}{2} \left( \frac{d}{dz} B_z \right) + \frac{r^3}{16} \left( \frac{d^3}{dz^3} B_z \right)$$
‘soft-edge’ solenoid – nonlinear effects

In tracking simulations the first nonlinear focusing term, $\Delta F \sim O(r^2)$ is also included:

$$\Phi \equiv \frac{1}{F} \approx \left( \frac{e}{2 pc} \right)^2 \left( \int B^2 ds + \frac{r^2}{2} \int B'^2 ds \right) \approx L \left( \frac{eB_0}{2 pc} \right)^2 \left( 1 + \frac{r^2}{3aL} \right)$$

Nonliner focusing at $r = 20$ cm for 1 m long solenoid with 25 cm aperture radius

$$\frac{\Delta F}{F} \approx \frac{r^2}{2 \int B'^2 ds} \approx \frac{r^2}{3aL} \quad L/r=4, r\approx 0.8a \rightarrow 0.07$$
Ionization cooling due to energy loss (-\(\Delta p\)) in a thin absorber followed by immediate re-acceleration (\(\Delta p\)) can be described as:

\[
\Delta \theta_\perp = -\theta_\perp \frac{\Delta p}{p}
\]

The corresponding canonical transfer matrix can be written as

\[
M_{abs} = K \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 - \frac{\Delta p}{p} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 - \frac{\Delta p}{p}
\end{bmatrix} K^{-1}
\]

\[
k = \frac{eB_z}{pc}
\]

\[
\hat{x} = Kx
\]

\[
\hat{x} \equiv \begin{bmatrix}
x \\
p_x \\
y \\
p_y \\
\theta_x \\
\theta_y
\end{bmatrix}
\]

\[
x \equiv \begin{bmatrix}
x \\
p_x \\
y \\
p_y \\
\theta_x \\
\theta_y
\end{bmatrix}
\]
Final cooling – initial beam parameters

\[ p = 287 \text{ MeV/c} \]

<table>
<thead>
<tr>
<th>after helical cooling channel</th>
<th>( \varepsilon_{\text{rms}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>normalized emittance: ( \varepsilon_x/\varepsilon_y )</td>
<td>mm·mrad</td>
</tr>
</tbody>
</table>
| longitudinal emittance: \( \varepsilon_l \)  
\( (\varepsilon_l = \sigma_{\Delta p} \sigma_z/m_\mu c) \)  
momentum spread: \( \sigma_{\Delta p/p} \)  
bunch length: \( \sigma_z \) | mm | 7 |
|                              | mm             | 0.03 |
|                              | mm             | 100  |
Solenoid cell, no absorber ($\Delta \phi = \pi$) – particle tracking
Solenoid cell with absorber ($\Delta \phi = \pi$) – particle tracking
Absorber – G4BL model

50 cm long box of gaseous Hydrogen at 200 atm

\[ E = 200 \text{ MeV} \]
\[ p = 287 \text{ MeV/c} \]
Absorber – G4BL simulation

Initial test beam - muons at $p = 287$ MeV/c

parallel beam with no momentum spread
Solenoid cell – resonant vs non-resonant

L[cm]=130  
L[cm]=80

B[kG]=38.1  
B[kG]=-34.3

L[cm]=130  
L[cm]=80

B[kG]=32.9  
B[kG]=-32.1
Quadrupole cell – G4BL view
Solenoid cell – G4BL view
Solenoid cell – G4BL view
Solenoid cell – G4BL trajectories
Summary

Present status…

Prototype PIC lattices - quadrupole and solenoid channels

- Lattices with absorbers studied via transfer matrix code
- Building and testing G4BL tools

Initial G4BL study of a solenoid channel

Future work…

- G4beamline simulation of a quadrupole channel with absorbers
- G4beamline simulation of a solenoid channel with absorbers
- G4beamline simulation with absorbers followed by RF cavities
- Emittance implementation – ecalc9